

Combinatorial Games

What is the mathematical theory behind combinatorial games?

- strategies of 2-player games which players take turns to achieve a defined winning condition
- 2-player games that have neither chance elements nor concealed information

Let's look at two games.

+ Game of 50

+ Game of Take Away

Game of 50

How to play:

Each player takes turns to select a number (1,2,3,4,5,6) as an addend to the sum of the earlier numbers.

The player who is the first to arrive at '50' is the winner. (after adding the number that he/she has selected)

Is there a winning strategy?

Does my strategy depend on my 'starting' number?

Does my strategy depend on the types of numbers chosen? (eg. odd/even, factors)

Does the order of players matter?

If my opponent chooses '1', does it mean that you will definitely lose the game?

Are there 'target' numbers?

Are there numbers to avoid?

Are there numbers to 'make'?

Are there numbers that will lead you to win the game?

Game of Take Away

How to play:

Each player grabs a handful of counters to form 2 piles of counters.

Each player takes turns to remove any number of counters from one of the groups of counters.

(A or B, but not both at once)

The player who takes the last counter or last group of counters is the winner.

Is there a winning strategy?

Does my strategy depend on the number of counters at the start?

Does my strategy depend on the number of counters taken at the start? (eg. odd/even number of counters)

Does the order of players matter?

Now that you know the winning strategies, can you create similar problems to play with your friends?

Winning strategy for Game of 50

You have to reach '43' first before your opponent.

In order to reach '43', you have to reach '36'.

Working backwards, you have to be the first to reach these numbers:

43, 36, 29, 22, 15, 8, 1

Winning strategy for Game of Take Away

To win, make sure that both groups have the same number of counters after each of your turn.

If you have 3 groups instead of 2, remove all the counters in 1 group when the other 2 groups have an equal number of counters.

Never do this to leave unequal groups, allowing your opponent to then create 2 equal groups of counters.