

Euler's Polyhedral Formula

1. Who is Euler? What is his contribution to the study of solid figures?
2. What is a polygon? What is a polyhedron?
Define these two solid figures in terms of their bases and faces.
In geometry, what is a platonic solid?
3. What is Euler's Polyhedral Formula?
Use 5 platonic solids to illustrate this formula.
Show the relationship between the edges, faces and vertices of polyhedral.
(Include hand-drawn diagrams in your tabulation of findings)
4. What is the Euler Characteristic for any convex polyhedron's surface?
How many hexagons are found on a soccer ball?

Fill in the blanks:

Every pentagon has _____ vertices and every such shape is shared by _____ faces.

Every hexagon has _____ vertices and every such shape is shared by _____ faces.

5. Using the information in part (ii) and using N for the number of hexagons and L for the number of pentagons, use Euler's Polyhedral Formula and Euler Characteristic of a sphere to derive the number of pentagons on a soccer ball.
What is V (vertices) ?
What is E (edges) ?
What is F (faces) ?
6. Using suitable materials create a 3-dimensional model of a soccer ball.

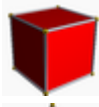
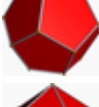
Answers:

2. Polygon : A closed shape that is made up of points joined by straight lines in 2-dimension.

Polyhedron : A closed solid object whose surface is made up of a number of polygonal faces.

A platonic solid has faces that are congruent regular polygons with the same number of faces meeting at each vertex. Its edges, vertices and angles are also congruent.

3. *Euler's Polyhedral Formula states that* Euler's formula tells us that the number of vertices minus the number of edges plus the number of faces is equal to two, i.e. $V - E + F = 2$

Name	Image	Vertices V	Edges E	Faces F	Euler characteristic: $V - E + F$
Tetrahedron		4	6	4	2
Hexahedron or cube		8	12	6	2
Octahedron		6	12	8	2
Dodecahedron		20	30	12	2
Icosahedron		12	30	20	2

4. The Euler Characteristic for any convex polyhedron is 2.

20 hexagons on a soccer ball.

Every pentagon has 5 vertices and every such shape is shared by 3 faces.

Every hexagon has 6 vertices and every such shape is shared by 2 faces.

$$\text{Euler Characteristic} = 2 = V - E + F \quad (0.5 \text{ mark})$$

$$= \frac{5L + 6N}{3} - \frac{5L + 6N}{2} + (L + N) \quad (0.5 \text{ mark})$$

$$= \frac{L}{6}$$

$$\text{To satisfy the equation, } L = 12. \quad (0.5 \text{ mark})$$