WALK THE DOG

The Scenario

- Jenny runs a very profitable business in the city: she walks dogs for people who don't have time to do it themselves. It's a perfect job for her since she loves animals and also loves to be outside.

- Twelve of Jenny's clients live in Anderson Apartments. Each has one dog.

- The apartment numbers of her 12 clients are: 3A, 4B, 4D, 5C, 6A, 6B, 6E, 8H, 9C, 9J, 10E and 12U

- Jenny can walk any number of dogs at one go if the situation the dogs and the owners are fine with the arrangement.

The Objective

- Jenny aims to find out the fewest number of trips she must complete to exercise all the dogs in Anderson Apartments for which she is responsible.

The Constraints

- Some dogs must be kept apart as they don't get along with one another.
- Other dogs must be kept apart as their owners aren't particularly fond of each other.
- Asterisks (X) in the chart indicate by apartment number those pairs of animals that must be kept apart

	3A	4B	4D	5C	6A	6B	6E	8H	9C	9J	10E	12U
3A	-	X	X			X			X		X	
4B	X	-	X	X	X			X		X		X
4D	X	X	-	X			X					X
5C		X	X	-	X		X	X				
6A		X		X	-	X			X	X		
6B	X				X	-	X					X
6E			X	X		X	-	X			X	
8H		X		X			X	-	X			
9C	X				X			X	-	X		X
9J		X			X				X	-	X	
10E	X						X			X	-	X
12U		X	X			X			X		X	-

Some Examples

Referring to the chart:

- Dogs from Apartment 3A and 4B **cannot** be placed on the same trip
- Dogs from Apartment 6B and 4D **can** be placed on the same trip

Jenny must …

- Bring the dogs out in groups!
- Try to have as few groups as possible

The maximum number of groups is 12 (each dog is a “group” by itself)

The minimum number groups is 1 (but is it possible?!)

Question 1

- For each of the dog in the 12 apartments, list the number of dogs that it cannot be walked together with in the table given.

Completing the Table …

	3A	4B	4D
3A	-	X	X
4B	X	-	X
4D	X	X	-
5C		X	X
6A		X	
6B	X		
6E			X
8H		X	
9C	X		
9J		X	
10E	X		
12U		X	X

Dog in Apartment	3A	4B	4D
No. of restrictions	5		

Question 2

- Which dog has the highest number of restrictions?

(restrictions: number of dogs that cannot be walked together with the one in question)

Question 3

- Jenny can minimize the number of trips if she can find as many dogs as she can to walk with the dog identified in Question 2.

- List the dogs that can be walked with the dog identified in Question 2.

Some more examples about the chart

	3A	4B	4D	5C	6A	6B	6E	8H	9C	9J	10E	12U
3A	-	X	X			X			X		X	

Referring to the chart:

Dogs from Apartment **3A can** be walked with dogs from Apartments **5C**, **6A**, **6E**, **8H**, **9J**, **12U.**

Question 4

- Can we walk all the dogs listed in Question 2 and Question 3 at the same time?

Question 5

- Find an optimal (most favourable) combination of dogs that can be walked together with the dog identified in Question 2. **Explain** your steps.

Hints for Question 5

- (Dog in Question 2) **can** go with (Dogs in Question 3)
- Taking a closer look at these dogs in Question 3, we know that these dogs **cannot** be walked together at the same time.
- Which dog can we take out from this group so that the maximum number of dogs can be walked together?
- What is the optimal combination then? (Which are the dogs that can walk together)

More Hints for Question 5

- 4B can go with 6B, 6E, 9C, 10E
- Taking a closer look at these 4 dogs (6B,6 10E):

	6B	6E	9C	10E
6B	-	X		
6E	X	-		X
9C			-	
10E		X		-

- Which dog has the highest number of restriction? You should remove it from the group of 4 dogs.
- What is then your optimal combination? (Which are the dogs that can walk together)

Question 6

- Do not include the dogs that have already been identified in the combination in Question 5. (ie – you have identified the first group of dogs to be taken out for a walk)

- From the remaining number of dogs, find the one that now has the highest number of restrictions among all the reminding dogs.

Hints for Question 6

- There are 12 dogs in total, after you have identified the first group of dogs in Question 5, which are the dogs that are left?
- Relook at the chart on Page 1 of your worksheet, find the new restrictions for all the dogs that are left.
- Identify the dog that has the highest number of restrictions
- If there are more than 1 answer, choose only 1 dog as your answer

Question 7

- By again considering the optimal combination of dogs that can be walked together with the dog identified in Question 6, find a combination that would allow Jenny to make the fewest number of trips with all the dogs.

Hints for Question 7

- Which are the dogs that **can** go with the dog that is identified in Question 6?
- Taking a closer look at these dogs, we know that **cannot** be walked together at the same time again! (Similar approach as in Question 5)
- Which dog can we take out from this new group so that the maximum number of dogs can be walked together?
- What is the optimal combination for this 2nd group of dogs then? (Which are the dogs that can walk together)

Question 8

- What is the fewest number of trips Jenny must make?

Hints for Question 8

- You will have 2 groups of dogs by now, this will imply that Jenny will need to take at least 2 trips
- How many more dogs are left?
- Can we group them together and walk them using another trip?
- If not, how many more trips do we need for the remaining dogs?

Question 9

- Find another combination of dogs she can take that would allow her to still make the fewest number of trips.

Hints for Question 9

- Go back to Question 6, is there another answer for Question 6?
- If so, use this answer to come out with another combination of dogs.

Question 10

- The dog owners are willing to pay more if their dogs are walked in a smaller group since that means their dog gets more attention. Jenny thus charges her fees as follows.

No. of dogs walked together	Fees for each dog
5	$4
4	$5
3	$6
2	$7
1	$8

Question 10

- Which combination (Question 7 or Question 9) will allow Jenny to make the most money with the fewest number of trips made? How much more?

Solutions

Solution - Qn 1

Dog in Apartment	3A	4B	4D	5C	6A	6B	6E	8H	9C	9J	10E	12U
No. of restrictions	5	7	5	5	5	4	5	4	5	4	4	5

Detailed Solution – Qn1

	3A	4B	4D	5C	6A	6B	6E	8H	9C	9J	10E	12U
3A	-	X	X			X			X		X	
4B	X	-	X	X	X			X		X		X
4D	X	X	-	X			X					X
5C		X	X	-	X		X	X				
6A		X		X	-	X			X	X		
6B	X				X	-	X					X
6E			X	X		X	-	X			X	
8H		X		X			X	-	X			
9C	X				X			X	-	X		X
9J		X			X				X	-	X	
10E	X						X			X	-	X
12U		X	X			X			X		X	-

Dog in Apartment	3A	4B	4D	5C	6A	6B	6E	8H	9C	9J	10E	12U
No. of restrictions	5	7	5	5	5	4	5	4	5	4	4	5

Solution - Qn 2

Dog in Apartment	3A	4B	4D	5C	6A	6B	6E	8H	9C	9J	10E	12U
No. of restrictions	5	**7**	5	5	5	4	5	4	5	4	4	5

The dog in Apartment **4B** has the highest number of restrictions.

Solution - Qn 3

	3A	4B	4D	5C	6A	6B	6E	8H	9C	9J	10E	12U
4B	X	-	X	X	X			X		X		X

The dogs that can be walked with dog **4B** are the dogs from **6B, 6E, 9C, 10E**

Solution - Qn 4

No, we cannot.

Solution - Qn 5

	4B	6B	6E	9C	10E
4B					
6B		-	X		
6E		X	-		X
9C				-	
10E			X		-

From the above chart, we can see that **6E** has the most restrictions. If **6E** is taken out of the group, the other dogs can be taken out for the same walk.

Therefore, the optimal combination is **4B, 6B, 9C and 10E.**

Jenny will be walking the 12 dogs in the following groups:

Group 1 – 4B, 6B, 9C, 10E

The remaining dogs are still unsorted.

Solution - Qn 6

Dog in Apartment	3A	4B	4D	5C	6A	6B	6E	8H	9C	9J	10E	12U
Answer (as in Question 1)	5	7	5	5	5	4	5	4	5	4	4	5
New number of restrictions	1	-	**4**	**4**	2	-	3	2	-	1	-	1

The dogs in Apartment **4D or 5C** has the highest number of restrictions.

Detailed Solution – Qn6

	3A	4D	5C	6A	6E	8H	9J	12U
3A	-	X						
4D	X	-	X		X			X
5C		X	-	X	X	X		
6A			X	-			X	
6E		X	X		-	X		
8H			X		X	-		
9J				X			-	
12U		X						-

Dog in Apartment	3A	4D	5C	6A	6E	8H	9J	12U
No. of restrictions	1	**4**	**4**	2	3	2	1	1

The dogs in Apartments **4D or 5C** has the highest number of restrictions.

Jenny will be walking the 12 dogs in the following groups:

Group 1 – 4B, 6B, 9C, 10E

Either 4D or 5C → Group 2 - We can place either 4D or 5C in this group, but not both of them

The remaining dogs are still unsorted.

Solution - Qn 7 and 9 (Part 1a)

What do we have thus far …

- First group of dogs - **4B, 6B, 9C, 10E** (Answer is from Question 5)
- For the remaining number of dogs, the dogs in Apartments **4D or 5C** has the highest number of restrictions.

Using the dog from Apartment **4D** as the starting point to find the 2nd group of dogs to be taken for walks …

	3A	4D	5C	6A	6E	8H	9J	12U
4D	X	-	X		X			X

The dogs that can be walked with dog **4D** are the dogs from **6A, 8H, 9J.**

Jenny will be walking the 12 dogs in the following groups:

Group 1 – 4B, 6B, 9C, 10E

Either 4D or 5C

Group 2 - We can place either 4D or 5C in this group, but not both of them

We decided to place 4D in group 2

The remaining dogs are still unsorted.

Solution - Qn 7 and 9 (Part 1b)

	4D	6A	8H	9J
4D	-			
6A		-		X
8H			-	
9J		X		-

From the chart, we can see that **6A** and **9J** cannot be taken out for the same walk.

Solution - Qn 7 and 9 (Part 1c)

Jenny will be walking the 12 dogs in the following groups:

Group 1 – 4B, 6B, 9C, 10E

Either 4D or 5C → Group 2 - We can place either 4D or 5C in this group, but not both of them

If 4D, either 6A or 9J → If we place, 4D in group 2, we have to choose between 6A and 9J

Group 2 - 4D, 6A, 8H

The remaining dogs are still unsorted.

Dog in Apartment	3A	4D	5C	6A	6E	8H	9J	12U
No. of restrictions	1	4	4	2	3	2	1	1

Relooking at the table on the number of restrictions from solution (Qn 6), we notice that **6A** has 1 more restriction than **9J**, therefore we will consider **6A** first.

Therefore, the optimal combination for the 2nd group of dogs is **4D, 6A, 8H.**

Solution - Qn 7 and 9 (Part 1d)

	3A	5C	6E	9J	12U
3A	-				
5C		-	X		
6E		X	-		
9J				-	
12U					-

From the chart, we can see that **5C** and **6E** cannot be taken out for the same walk.

Therefore, the optimal combination for the 3rd group of dogs is **3A, 5C, 9J, 12U** **OR** **3A, 6E, 9J, 12U**

Solution - Qn 7 and 9 (Part 1e)

The optimal combination for Jenny to make the least number of trips is:

Group 1 – **4B, 6B, 9C, 10E** (Answer is from Question 5)
Group 2 – **4D, 6A, 8H** (We chose 4D instead of 5C and 6A instead of 9J)
Group 3 – **3A, 5C, 9J, 12U** (We chose 5C instead of 6E)
Group 4 – **6E**

OR

Group 1 – **4B, 6B, 9C, 10E** (Answer is from Question 5)
Group 2 – **4D, 6A, 8H** (We chose 4D instead of 5C)
Group 3 – **3A, 6E, 9J, 12U** (We chose 6E instead of 5C)
Group 4 – **5C**

Solution - Qn 7 and 9 (Part 2)

Other possible answers:

Group 1 – **4B, 6B, 9C, 10E** (Answer is from Question 5)
Group 2 – **4D, 9J, 8H** (We chose 4D instead of 5C and 9J instead of 6A)
Group 3 – **3A, 5C, 12U** (We chose 5C instead of 6E)
Group 4 – **6E, 6A**

Group 1 – **4B, 6B, 9C, 10E** (Answer is from Question 5)
Group 2 – **3A, 5C, 9J, 12U** (We chose 5C instead of 4D)
Group 3 – **4D, 6A, 8H**
Group 4 – **6E**

Group 1 – **4B, 6B, 9C, 10E** (Answer is from Question 5)
Group 2 – **3A, 5C, 9J, 12U** (We chose 5C instead of 4D)
Group 3 – **4D, 8H**
Group 4 – **6E, 6A**

Solution - Qn 8

4 trips

Solution - Qn 10

For the combination:
Group 1 - 4B, 6B, 9C, 10E
Group 2 - 4D, 6A, 8H
Group 3 - 5C, 3A, 9J, 12U
Group 4 - 6E

Fee is 8 × \$5 + 3 × \$6 + \$8 = **\$66**

For the combination:
Group 1 - 4B, 6B, 9C, 10E
Group 2 - 4D, 9J, 8H
Group 3 - 5C, 3A, 12U
Group 4 - 6A, 6E

Fee is 4 × \$5 + 6 × \$6 + 2 × \$7 = **\$70**

The combination that will allow her to make more money is
Group 1 - 4B, 6B, 9C, 10E
Group 2 - 4D, 9J, 8H
Group 3 - 5C, 3A, 12U
Group 4 - 6A, 6E

The End